MATH 20D Spring 2023 Lecture 6.

Intergrating Factors, Mixing, and Cooling

Outline

1. Integrating Factors
2. Applications of First Order ODE's

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- You will be give a table of standard integrals for your midterms and final exam.

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to obtain $e^{2t}y'(t) + 2e^{2t}y(t) = 2e^{t}$. So $\frac{d}{dt}(e^{2t}y(t)) = 2e^{t}$.

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Step 3: Integrate and multiply by e^{-2t} to obtain the **general solution**

$$y\colon \mathbb{R} \to \mathbb{R}, \qquad y(t) = 2e^{-t} + Ce^{-2t}$$

Example

Let $0 < m < 1$ be constant. Show the general solution to the ODE

$$\frac{dy}{dx} + \frac{m\cos(x)}{2(1+m\sin(x))}y = \sqrt{1-m\sin(x)}$$

can be expressed in the form

$$y\colon \mathbb{R} \to \mathbb{R}, \quad y(t) = \frac{1}{\sqrt{1+m\sin(t)}}\left(C + \int_0^t \sqrt{1-m^2\sin^2(x)}dx\right).$$

where C is a constant.

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- After 10 minutes a gushing leak develops and the rate of outflow from the tank increases to 12L/min

(b) Determine the volume of nitric acid in the tank after 10 minutes after the leak develops. *Express your answer to the nearest 0.01L*

Part (a)

The general solution to the ODE has the form

$$N = 36 - Ae^{-t/30}$$

substituting $N(0) = 180 \cdot \frac{1}{10} = 18$ we find that $A = 18$. Hence $N(10) = 23.1024$.

Part (b)

The differential equation takes the form

$$\frac{dN}{dt} + \frac{2N}{30 - t} = 1.2$$

The integrating factor is then given by

$$\mu(t) = \exp\left(\int \frac{2dt}{30 - t}\right) = \exp(-2\log|30 - t|) = \exp(\log|30 - t|^{-2})$$

So $\mu(t) = (30 - t)^{-2}$ for all $t \leqslant 30$.